

# Crystal geometry equations for XRD

## D-spacings and Miller indices

Symmetry:

Cubic:

$$\frac{1}{d^2} = \frac{h^2 + k^2 + l^2}{a^2}$$

Tetragonal:

$$\frac{1}{d^2} = \frac{h^2 + k^2}{a^2} + \frac{l^2}{c^2}$$

Hexagonal:

$$\frac{1}{d^2} = \frac{4}{3} \left( \frac{h^2 + hk + k^2}{a^2} \right) + \frac{l^2}{c^2}$$

Rhombohedral:

$$\frac{1}{d^2} = \frac{(h^2 + k^2 + l^2) \sin^2 \alpha + 2(hk + kl + hl)(\cos^2 \alpha - \cos \alpha)}{a^2(1 - 3 \cos^2 \alpha + 2 \cos^3 \alpha)}$$

Orthorhombic:

$$\frac{1}{d^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}$$

Monoclinic:

$$\frac{1}{d^2} = \frac{1}{\sin^2 \beta} \left( \frac{h^2}{a^2} + \frac{k^2 \sin^2 \beta}{b^2} + \frac{l^2}{c^2} - \frac{2hl \cos \beta}{ac} \right)$$

Triclinic:

$$\frac{1}{d^2} = \frac{1}{V^2} (S_{11}h^2 + S_{22}k^2 + S_{33}l^2 + 2S_{12}hk + 2S_{23}kl + 2S_{13}hl)$$

where

$$S_{11} = b^2 c^2 \sin^2 \alpha$$

$$S_{22} = a^2 c^2 \sin^2 \beta$$

$$S_{33} = a^2 b^2 \sin^2 \gamma$$

$$S_{12} = abc^2(\cos \alpha \cos \beta - \cos \gamma)$$

$$S_{23} = a^2 bc(\cos \beta \cos \gamma - \cos \alpha)$$

$$S_{13} = ab^2 c(\cos \gamma \cos \alpha - \cos \beta)$$

## Unit cell volume

Symmetry:

Cubic:

$$V = a^3$$

Tetragonal:

$$V = a^2 c$$

Hexagonal:

$$V = \frac{\sqrt{3} a^2 c}{2}$$

Rhombohedral:

$$V = a^3 \sqrt{1 - 3 \cos^2 \alpha + 2 \cos^3 \alpha}$$

Orthorhombic:

$$V = abc$$

Monoclinic:

$$V = abc \sin \beta$$

Triclinic:

$$V = abc \sqrt{1 - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma + 2 \cos \alpha \cos \beta \cos \gamma}$$

## Angle $\phi$ between planes $(h_1 k_1 l_1)$ and $(h_2 k_2 l_2)$

Symmetry:

Cubic:

$$\cos \phi = \frac{h_1 h_2 + k_1 k_2 + l_1 l_2}{\sqrt{(h_1^2 + k_1^2 + l_1^2)(h_2^2 + k_2^2 + l_2^2)}}$$

Tetragonal:

$$\cos \phi = \frac{\frac{h_1 h_2 + k_1 k_2}{a^2} + \frac{l_1 l_2}{c^2}}{\sqrt{\left(\frac{h_1^2 + k_1^2}{a^2} + \frac{l_1^2}{c^2}\right) \left(\frac{h_2^2 + k_2^2}{a^2} + \frac{l_2^2}{c^2}\right)}}$$

Hexagonal:

$$\cos \phi = \frac{h_1 h_2 + k_1 k_2 + \frac{1}{2}(h_1 k_2 + h_2 k_1) + \frac{3a^2}{4c^2} l_1 l_2}{\sqrt{\left(h_1^2 + k_1^2 + h_1 k_1 + \frac{3a^2}{4c^2} l_1^2\right) \left(h_2^2 + k_2^2 + h_2 k_2 + \frac{3a^2}{4c^2} l_2^2\right)}}$$

Rhombohedral:

$$\cos \phi = \frac{a^4 d_1 d_2}{V^2} [\sin^2 \alpha (h_1 h_2 + k_1 k_2 + l_1 l_2) + (\cos^2 \alpha - \cos \alpha) (k_1 l_2 + k_2 l_1 + l_1 h_2 + l_2 h_1 + h_1 k_2 + h_2 k_1)]$$

Orthorhombic:

$$\cos \phi = \frac{\frac{h_1 h_2}{a^2} + \frac{k_1 k_2}{b^2} + \frac{l_1 l_2}{c^2}}{\sqrt{\left(\frac{h_1^2}{a^2} + \frac{k_1^2}{b^2} + \frac{l_1^2}{c^2}\right) \left(\frac{h_2^2}{a^2} + \frac{k_2^2}{b^2} + \frac{l_2^2}{c^2}\right)}}$$

Monoclinic:

$$\cos \phi = \frac{d_1 d_2}{\sin^2 \beta} \left[ \frac{h_1 h_2}{a^2} + \frac{k_1 k_2 \sin^2 \beta}{b^2} + \frac{l_1 l_2}{c^2} - \frac{(l_1 h_2 + l_2 h_1) \cos \beta}{ac} \right]$$

Triclinic:

$$\cos \phi = \frac{d_1 d_2}{V^2} [S_{11} h_1 h_2 + S_{22} k_1 k_2 + S_{33} l_1 l_2 + S_{23} (k_1 l_2 + k_2 l_1) + S_{13} (l_1 h_2 + l_2 h_1) + S_{12} (h_1 k_2 + h_2 k_1)]$$