Single-crystal elasticity of grossular- and almandine-rich garnets to 11 GPa by Brillouin scattering

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[1] The high-pressure elasticity of grossular-rich Grs87And9Pyp2Alm2 and almandine-rich Alm72Pyp20Sp83Grs3And2 natural garnet single crystals were determined by Brillouin scattering to 11 GPa in a diamond anvil cell. The experiments were carried out using a 16:3:1 methanol-ethanol water mixture as pressure medium. The aggregate moduli as well as their pressure derivatives were obtained by fitting the data to Eulerian finite strain equations. The inversion yields K0 = 165.0 ± 0.9 GPa, G0 = 104.2 ± 0.3 GPa, (dK/G0/dP)0 = 3.8 ± 0.2, and (dG/G0/dP)0 = 1.1 ± 0.1 for the grossular-rich composition and K0 = 174.9 ± 1.6 GPa, G0 = 95.6 ± 0.5 GPa, (dK/G0/dP)0 = 4.7 ± 0.3, and (dG/G0/dP)0 = 1.4 ± 0.1 for the almandine-rich garnet. Both individual and aggregate elastic moduli of the two garnets define nearly linear modulus pressure trends. The elastic anisotropy of the garnets increases weakly in magnitude with compression. Isothermal compression curves derived from our results are generally consistent with static compression data under hydrostatic conditions, and the effects of nonhydrostaticity on previous diffraction data can be identified. The pressure derivatives obtained here are generally lower than those reported in high-pressure polycrystalline ultrasonic elasticity studies. In combination with earlier Brillouin scattering data for pyrope, our results allow us to constrain the effect on elastic moduli of Fe2+-Mg2+ substitution in pyrope-almandine, Ca2+-Mg2+ in pyrope-grossular, and Fe3+-Al3+ substitution in andradite-grossular at high pressures. This new data set thus allows us to place improved constraints on the compositional dependence of seismic velocities in the rocks of the upper mantle. INDEX TERMS: 3909 Mineral Physics: Elasticity and anelasticity; 3924 Mineral Physics: High-pressure behavior; 3934 Mineral Physics: Optical, infrared, and Raman spectroscopy; 3919 Mineral Physics: Equations of state; KEYWORDS: elasticity, garnet, high pressure, diamond anvil cell, sound velocity, Brillouin scattering


1. Introduction

[2] Garnets are abundant minerals in the igneous and metamorphic rocks of the Earth’s crust as well as important constituents of the mantle. Natural garnets in peridotites at pressures greater than ~1.5 GPa (45 km depth) in the Earth’s mantle are pyrope-rich with variable amounts of grossular and almandine [Rickwood et al., 1968; Lee, 2003]. One of the major phase changes expected under upper mantle conditions is the dissolution of pyroxene into the garnet structure producing Al-deficient garnets (majorite)[Ringwood, 1967, 1991; Fei and Bertka, 1999]. Mantle mineralogical models such as pyroille and picoglitte contain garnet volume fractions at low pressures of ~15% and ~22% respectively and these increase to 40% or more vol. fraction of garnet-majorite at transition zone conditions [Ringwood, 1991; Fei and Bertka, 1999]. For MORB compositions, the garnet fraction ranges from ~25% at conditions corresponding to the top of the upper mantle to ~90% majorite-garnet at transition zone conditions [Irifune and Ringwood, 1993]. Understanding the elastic properties of garnets is thus essential to the interpretation of regional seismic profiles of the upper 660 km of the Earth’s interior [Duffy and Anderson, 1989; Weidner and Wang, 2000]. In addition, the effect of compositional changes on elastic properties is important for interpreting lateral variations in seismic velocity imaged by seismic tommography. The effects of Mg2+-Ca2+ and Mg2+-Fe2+ substitution on the elastic properties of mantle minerals at upper mantle conditions are not well constrained [Karato and Karki, 2001]. Such data are necessary, for example, in interpreting seismic and geodynamic studies of the continental lithosphere (tectosphere) in terms of thermal and chemical properties of the region [Jordan, 1978; Forte and Perry, 2000; Lee, 2003]. The
elastic properties of garnets are also important for modeling seismic velocities in the lower continental crust [Jackson et al., 1990; Morozov et al., 2003].

The silicate garnet group $X_3Y_2(SiO_4)_3$ includes a series of isomorphous species with space group $Ia3d$, where the eight coordinated X-site houses cations such as $Ca^{2+}$, $Mg^{2+}$, $Fe^{2+}$, or $Mn^{2+}$ and the six coordinated Y-site incorporates $Al^{3+}$, $Fe^{3+}$, and $Cr^{3+}$ among others. The structure consists of alternating SiO$_4$ tetrahedra and YO$_6$ octahedra which share corners to form a three-dimensional network. The elastic tensor for cubic garnets consists of three independent elastic stiffness coefficients: $C_{11}$, $C_{12}$, and $C_{44}$. The bulk moduli of garnets at ambient pressure can be described by elasticity systematics [Bass, 1986; Wang and Ji, 2001], and can also be related to the relative compressibility of different structural units, especially the dodecahedral site [Milman et al., 2001]. However, a complete picture of the compositional dependence on the shear modulus, individual $C_{ij}$, and all pressure derivatives is not yet available [e.g., O’Neill et al., 1989; Wang and Ji, 2001].

Owing to their wide compositional range and the ready availability of good quality specimens, the elastic properties of garnets have been studied far more extensively than any other mineral group [e.g., Wang and Ji, 2001]. Numerous X-ray diffraction studies at high pressure have reported the isothermal bulk modulus, $K_{T0}$, and its pressure derivative, $(\partial K_{T0}/\partial P)_{T0}$ [Knittle, 1995]. Measurements of the elastic tensor (and corresponding aggregate bulk and shear moduli) at ambient and high pressures have been reported using ultrasonic interferometry or resonant ultrasound spectroscopy as well as light scattering methods such as Brillouin spectroscopy. At ambient pressure, there is good agreement for both individual and aggregate elastic properties among various studies using different experimental techniques. At high pressures, however, there are disagreements of up to 50% or more in reported pressure derivatives of the bulk and shear modulus for a given composition. For example, reported pressure derivatives for the bulk modulus of grossular range from 4.5 [Weaver et al., 1976] to 6.1 [Olijnyk et al., 1991]. Similarly values of $K_{T0}’=(\partial K_{T0}/\partial P)_{T0}$ and $G_0=(\partial G_0/\partial P)_0$ for majorite garnets differ by 60% and 36% in two reported studies [Gwanmesia et al., 1998; Sinogeikin and Bass, 2002]. The magnitude of these discrepancies greatly hinders efforts to make geological interpretations on the basis of seismic data [Sinogeikin and Bass, 2002].

In this study, we have carried out single-crystal elasticity measurements on grossular-rich ($Ca_3Al_2Si_3O_{12}$), and Fe-rich almandine-pyrope ($Fe(Mg)_3Al_2Si_3O_{12}$) garnets to pressures in excess of 11 GPa by Brillouin spectroscopy. We have paid careful attention to sources of systematic error (e.g., vignetting due to the limited angular opening of DAC, resulting in asymmetric Brillouin peak lineshapes), compositional heterogeneity, and have maintained hydrostatic conditions in the diamond anvil cell. Together with our detailed study on andradite [Jiang et al., 2004] and a previous Brillouin study on pyrope [Sinogeikin and Bass, 2000], we are able to constrain the effects of $Fe^{2+}$-$Mg^{2+}$, $Ca^{2+}$-$Mg^{2+}$, and $Fe^{3+}$-$Al^{3+}$ substitution on the elastic properties of garnets at high pressures. Grossular and almandine-rich garnets were chosen because these compositions have not been extensively studied using optical spectroscopy techniques [cf. Chai et al., 1997; Conrad et al., 1999]. Better characterization of such compositions is needed as natural garnets from mantle peridotites contain roughly 12–26 mol % almandine and 2–20 mol % grossular, in addition to 60–86 mol % pyrope [Rickwood et al., 1968; Lee, 2003]. Furthermore, we evaluate the cause of the wide variability of the reported pressure derivatives of the elastic moduli among garnets, and derive a consistent set of elastic properties based on optical spectroscopy (Brillouin scattering) studies.

2. Experimental Procedure

A grossular single crystal from Sierra de las Cruces, Coahuila, Mexico and a natural almandine single crystal of

![Figure 1](image-url)

**Figure 1.** (a) Brillouin spectra of grossular-rich garnet at room pressure and 3.2 GPa. (b) Brillouin spectra of almandine-rich garnet at room pressure and 10.8 GPa. Brillouin peaks from P-mode, S-mode, pressure medium (M-E-W), and diamond are labeled.

### Table 1. Result of Microprobe and Powder X-Ray Diffraction Analysis at Ambient Conditions

<table>
<thead>
<tr>
<th>Oxides, wt %</th>
<th>SiO$_2$</th>
<th>TiO$_2$</th>
<th>Al$_2$O$_3$</th>
<th>FeO</th>
<th>MnO</th>
<th>MgO</th>
<th>CaO</th>
<th>Lattice Parameter, Å</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grossular-rich garnet: Grs$<em>{97}$And$</em>{3}$Pyp$<em>{20}$Alm$</em>{3}$; other trace; $\rho = 3.605$ g/cm$^3$</td>
<td>40.67</td>
<td>0.39</td>
<td>21.35</td>
<td>3.69</td>
<td>0.08</td>
<td>0.67</td>
<td>36.93</td>
<td>11.886</td>
</tr>
<tr>
<td>Almandine-rich garnet: Alm$<em>{72}$Pyp$</em>{20}$Sp$<em>{5}$Grs$</em>{3}$And$_{2}$; other trace; $\rho = 4.132$ g/cm$^3$</td>
<td>37.76</td>
<td>0.01</td>
<td>21.69</td>
<td>33.53</td>
<td>1.39</td>
<td>4.99</td>
<td>1.79</td>
<td>11.536</td>
</tr>
</tbody>
</table>

*All iron assumed to be FeO.*
unknown origin were used. Both showed characteristic dodecahedral habits. Platelets parallel to the natural \{110\} faces were cut and then double-side polished with successively finer grits down to a final diamond paper of 1 \(\mu\text{m}\) particle size. The final sample thickness was approximately 30 \(\mu\text{m}\). The polished samples were checked using a polarizing microscope, and there was no evidence of zoning. The compositions of the samples used for our Brillouin measurements were determined by electron microprobe analysis. Ten points were analyzed across the samples, and they have equivalent compositions within experimental uncertainties. The densities at ambient conditions were determined by powder X-ray diffraction and chemical analysis (see Table 1).

[7] The polished platelets of grossular- and almandine-rich garnets were loaded into a modified Merrill-Basset diamond anvil cell with angular opening of 96° and compressed up to 11–12 GPa using a 16:3:1 methanol-ethanol-water mixture as a pressure-transmitting medium. In all experiments more than four ruby chips were placed around the sample as pressure calibrants. Pressure determination was performed by measuring the ruby fluorescence shift [Mao et al., 1986]. The fluorescence peaks did not show significant broadening over the whole pressure range. In order to allow for possible stress relaxation after each compression step, Brillouin measurements were carried out at least one day after pressure increase. The differences between pressure measured from different ruby chips around the samples never exceeded \(\pm 0.2\) GPa. Pressures measured before and after each Brillouin data set collection were always equivalent within mutual uncertainties.

[8] The samples were excited with a single mode vertically polarized neodymium vanadate laser (\(\lambda = 532.15\) nm) with a power of 150 mW. Brillouin spectra were measured using a six-pass Sandercock tandem Fabry-Perot interferometer in a forward symmetric scattering geometry, in which acoustic velocities, \(V\), can be determined without knowledge of the sample refractive index [Whitfield et al., 1976]:

\[
V = \frac{\Delta \nu_B \lambda_0}{2 \sin(\theta/2)},
\]

where, \(\lambda_0\) is the incident laser wavelength, \(\Delta \nu_B\) is the measured Brillouin frequency shift and \(\theta\) is the scattering angle external to the diamond cell (70° in this study). Details of the experimental setup are reported elsewhere [Speziale and Duffy, 2002].

[9] Special attention was paid to determine the scattering angle to within a few minutes of a degree using a reference laser beam. Therefore experimental errors due to scattering angles are neglected in our data analysis. The accuracy and reproducibility of our system was tested on standard single crystals of MgO and SrTiO\(_3\) with known velocities. A

### Table 2. Best-Fit Density, Elastic Constants, and Aggregate Moduli\(^a\)

<table>
<thead>
<tr>
<th>P, GPa</th>
<th>(\rho, \text{ g/cm}^3)</th>
<th>(C_{11}, \text{ GPa})</th>
<th>(C_{12}, \text{ GPa})</th>
<th>(C_{44}, \text{ GPa})</th>
<th>(K_0, \text{ GPa})</th>
<th>(G, \text{ GPa})</th>
<th>RMS, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>3.605</td>
<td>314.5(4)</td>
<td>91.5(5)</td>
<td>99.7(2)</td>
<td>165.8(5)</td>
<td>104.3(3)</td>
<td>21</td>
</tr>
<tr>
<td>1.6</td>
<td>3.640</td>
<td>323.5(12)</td>
<td>95.1(9)</td>
<td>100.7(5)</td>
<td>171.2(10)</td>
<td>105.9(8)</td>
<td>41</td>
</tr>
<tr>
<td>3.2</td>
<td>3.674</td>
<td>332.7(6)</td>
<td>99.9(6)</td>
<td>103.1(3)</td>
<td>177.5(6)</td>
<td>108.2(5)</td>
<td>27</td>
</tr>
<tr>
<td>4.3</td>
<td>3.697</td>
<td>337.4(7)</td>
<td>101.7(7)</td>
<td>102.1(3)</td>
<td>180.3(7)</td>
<td>108.1(6)</td>
<td>28</td>
</tr>
<tr>
<td>6.5</td>
<td>3.740</td>
<td>352.4(10)</td>
<td>104.9(5)</td>
<td>104.9(5)</td>
<td>189.8(9)</td>
<td>111.4(8)</td>
<td>35</td>
</tr>
<tr>
<td>7.9</td>
<td>3.767</td>
<td>354.(12)</td>
<td>105.8(4)</td>
<td>105.8(4)</td>
<td>191.9(9)</td>
<td>112.0(7)</td>
<td>33</td>
</tr>
<tr>
<td>9.4</td>
<td>3.798</td>
<td>360.(13)</td>
<td>116.6(9)</td>
<td>107.1(5)</td>
<td>201.3(10)</td>
<td>114.7(8)</td>
<td>37</td>
</tr>
<tr>
<td>10.7</td>
<td>3.823</td>
<td>377.(11)</td>
<td>121.3(8)</td>
<td>107.9(5)</td>
<td>206.7(9)</td>
<td>115.8(8)</td>
<td>32</td>
</tr>
</tbody>
</table>

\(^a\)Numbers in parentheses are 1-\(\sigma\) deviations in last digit. RMS, root mean square of the difference between observed and calculated velocities.

### Table 3. Thermodynamic Parameters Used for Adiabatic to Isothermal Conversion

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grossular</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal expansion (\alpha_T)</td>
<td>19.2 (\times 10^{-6}) K(^{-1})</td>
<td>Isaak et al. [1992]</td>
</tr>
<tr>
<td>Grüneisen parameter (\gamma_0)</td>
<td>1.22</td>
<td>calculated(^a)</td>
</tr>
<tr>
<td>Specific heat (C_P)</td>
<td>325.5(7) J/(mol ⋅ K)</td>
<td>Isaak et al. [1992]</td>
</tr>
<tr>
<td>((\partial K_0/\partial T)_P)</td>
<td>(-0.02) GPa K(^{-1})</td>
<td>Isaak et al. [1992]</td>
</tr>
<tr>
<td>Almandine</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grüneisen parameter (\gamma_0)</td>
<td>1.22</td>
<td>Soga [1967]</td>
</tr>
<tr>
<td>((\partial K_0/\partial T)_P)</td>
<td>(-0.0201) GPa K(^{-1})</td>
<td>Soga [1967]</td>
</tr>
<tr>
<td>((\partial \omega/\partial T)_P)</td>
<td>(5.24 \times 10^{-3}) K(^{-1})</td>
<td>Skinner [1966]</td>
</tr>
<tr>
<td>((\partial \omega/\partial T)_P)</td>
<td>(-0.0277) GPa K(^{-1})</td>
<td>calculated(^b)</td>
</tr>
</tbody>
</table>

\(^a\)Grüneisen parameter obtained as \(\gamma_0 = \alpha_T K_0/(\rho_0 C_P)\) using \(K_0\) and \(\rho_0\) of this study.

\(^b\)Calculated using \((\partial \omega/\partial T)_P \approx (\partial K_0/\partial T)_P(1 + \gamma_0 T) - K_0/(1 + \gamma_0 T)^2\)

\[\gamma_T + (\partial \omega/\partial T)_P\gamma_T\].
The only one varied during measurement. The azimuthal angle, \( \varphi \), was added in front of the collection lens. A half-wave plate in the incident light path was used to maximize the intensity of the Brillouin peaks [Jiang et al., 2004].

3. Results

For all Brillouin spectra, one quasi-longitudinal (P) and one quasi-transverse (S) acoustic mode were observed. Typical Brillouin spectra at ambient and high pressure are shown in Figure 1. The measured frequency shifts have been converted to velocity along the horizontal axis using equation (1). For both samples at each pressure, Brillouin measurement was performed for 36 directions at 5 degree intervals. Eulerian angles \((\theta_0, \varphi_0, \chi_0)\) relating the crystallographic coordinate system to the laboratory coordinate system were used to specify the crystal orientation and acoustic wave-vector direction. The azimuthal angle, \( \varphi_0 \), was the only one varied during measurement.

The velocity data were fitted to the Christoffel’s equation [Every, 1980] to retrieve the elastic constants \( (C_{11}, C_{12}, \text{and } C_{44}) \) and the three Eulerian angles describing the crystal orientation. A starting model based on literature values of \( C_{ij} \) and our measured density at ambient conditions was used for the room pressure velocity fitting, and the initial values for \( \theta_0, \varphi_0, \chi_0 \) were systematically varied until a satisfactory agreement between calculated and experimentally obtained velocities was attained. The recovered orientation indicated that the crystal plane was close to \( \{110\} \) as expected (see Table 2).

Despite the very low elastic anisotropy of these garnets \((\Lambda = 2C_{44}/(C_{11}-C_{12}) = 0.98) \) at ambient pressure for the almandine-pyrope garnet, we found that the crystal orientation could be readily recovered to within \( \pm 3 \) degrees. Densities at high pressures were initially estimated by using the Birch-Murnaghan equation of state, with ambient pressure values of the bulk modulus, \( K_0 \), and an initial guess for the pressure derivative of the bulk modulus, \( \partial K_0/\partial P \). High-pressure elastic constants were obtained by fitting each velocity curve using the calculated density and the \( C_{ij} \) values from the previous pressure as initial guesses. Errors in pressure determination in turn cause uncertainties in elastic moduli at high pressures by introducing errors in density. Our estimations show that the errors in elastic moduli would be 1.5 times the standard deviation 1-\( \sigma \) given in this study if this error in pressure determination is considered.

After the first round of fitting, the adiabatic aggregate bulk modulus and the aggregate shear modulus, \( G \), obtained from Voigt-Reuss-Hill average, were calculated. In the inversion of high-pressure moduli, an iterative procedure [Zha et al., 1996; Speziale and Duffy, 2002] was adopted. The calculated adiabatic bulk moduli \( K_S \) were fit to third-order Eulerian finite strain equation [Birch, 1978] to obtain \( K_{0S} \) and \( \partial K_S/\partial P \). These parameters were then converted to isothermal \( K_0 \) and \( \partial K_0/\partial P \), by applying thermodynamic relations:

\[
K_0 = K_{0S}/(1 + \alpha \gamma T) \tag{2}
\]

\[
(\partial K_0/\partial P)_0 \approx (1 + \alpha \gamma T)^{-1} [(\partial K_S/\partial P)_0 - \gamma T/(K_0/(\partial K_0/\partial T)_0)]. \tag{3}
\]

where \( \alpha \) is the volume thermal expansion coefficient and \( \gamma \) is the Grüneisen parameter (Table 3). The isothermal \( K_0 \) and \( \partial K_0/\partial P \) were then used to construct improved isothermal compression curves and the velocities at each pressure were refit. The above procedure was repeated and it converged after four iterations. It should be mentioned here that the initial values of \( K_0 \) and \( \partial K_0/\partial P \), which were used to calculate the initial densities at high pressures, did not affect final results, but only affect the number of iterations needed to achieve convergence.

![Figure 2. (a) Velocity data (symbols) and calculated velocity curves (lines) of grossular at room pressure and 4.3 GPa. (b) Velocity data and calculated velocity curves of natural almandine at room pressure and 10.8 GPa. Approximate directions [111], [100], and [110] are marked.](image)

### Table 4. Individual Elastic Constants and Pressure Derivatives of Grossular at Ambient Conditions

<table>
<thead>
<tr>
<th>Study</th>
<th>Composition</th>
<th>( C_{11} ), GPa</th>
<th>( C_{12} ), GPa</th>
<th>( C_{44} ), GPa</th>
<th>( \partial C_{11}/\partial P ), GPa</th>
<th>( \partial C_{12}/\partial P ), GPa</th>
<th>( \partial C_{44}/\partial P ), GPa</th>
<th>( P_{\text{max}} ), GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Grs98And, Ppy, Alm1</td>
<td>313.6(16)</td>
<td>90.7(7)</td>
<td>99.5(4)</td>
<td>6.1(2)</td>
<td>2.8(1)</td>
<td>0.9(1)</td>
<td>11</td>
</tr>
<tr>
<td>b</td>
<td>Grs90And2</td>
<td>321.7(8)</td>
<td>91.4(9)</td>
<td>104.6(4)</td>
<td>10(^{-4})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>Grs94And1Alm,Ppy</td>
<td>306.1(38)</td>
<td>88.7(38)</td>
<td>98.8(4)</td>
<td>10(^{-4})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Grs92And1Ppy</td>
<td>318.8(8)</td>
<td>92.1(7)</td>
<td>102.9(2)</td>
<td>10(^{-4})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{a}\)Numbers in parentheses are 1-\( \sigma \) deviations in last digit.

\(^{b}\)Studies are as follows: a, this study; b, Bass [1989]; c, Babuska et al. [1978]; d, Isaak et al. [1992].
Table 5. Aggregated Moduli and Pressure Derivatives of Grossular at Ambient Conditions a

<table>
<thead>
<tr>
<th>Study b</th>
<th>Composition</th>
<th>K_{SO}, GPa</th>
<th>K_{TO}, GPa</th>
<th>G_{SO}, GPa</th>
<th>(\partial K_{SO}/\partial P)_{TO}</th>
<th>(\partial K_{TO}/\partial P)_{TO}</th>
<th>(\partial G_{SO}/\partial P)_{TO}</th>
<th>P_{max}^*, GPa</th>
<th>Method c</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Grs 87, And, Pyp, Alm 1</td>
<td>165.0(9)</td>
<td>163.8(5)</td>
<td>104.2(3)</td>
<td>3.8(2)</td>
<td>3.9(2)</td>
<td>1.1(1)</td>
<td>11</td>
<td>BS</td>
</tr>
<tr>
<td>b</td>
<td>Grs 94, And 1</td>
<td>168.47</td>
<td>108.9(4)</td>
<td>10^{-4}</td>
<td>BS</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>c</td>
<td>Grs 87, And, Pyp 2</td>
<td>161.2(5)</td>
<td>120.6(4)</td>
<td>10^{-4}</td>
<td>U</td>
<td></td>
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</tr>
<tr>
<td>d</td>
<td>Grs 87, And, Pyp 1</td>
<td>167.8(7)</td>
<td>106.9(2)</td>
<td>10^{-4}</td>
<td>R</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e</td>
<td>Grs 87, And, Pyp 1</td>
<td>166.8 a d</td>
<td>108.9 d</td>
<td>5.46 d</td>
<td>10 BS</td>
<td>1.10</td>
<td>18.4 X</td>
<td>36.9 X</td>
<td>X</td>
</tr>
<tr>
<td>f</td>
<td>Grs 87, Pyp, Alm 1</td>
<td>173(2)</td>
<td>4.25 d</td>
<td>25</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Grs 87, Pyp, Alm 1</td>
<td>162(3)</td>
<td>5.45 d</td>
<td>25</td>
<td>X</td>
<td></td>
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</tr>
<tr>
<td>h</td>
<td>Grs 87, Pyp, Alm 1</td>
<td>165(4)</td>
<td>4.25 d</td>
<td>25</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i</td>
<td>Grs 87, Alm 1</td>
<td>169.3(12)</td>
<td>5.92(14)</td>
<td>36.9</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>j</td>
<td>Grs 87, Alm 1</td>
<td>175(1)</td>
<td>4.4 d</td>
<td>11.6</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Numbers in parentheses are 1-σ deviations in last digit.

Studies are as follows: a, this study; b, Verma [1960]; c, Wang and Simmons [1974]; d, Soga [1967]; e, Isaak and Graham [1976].

Table 6. Individual Elastic Constants and Pressure Derivatives of Almandine-Rich Garnet at Ambient Conditions a

<table>
<thead>
<tr>
<th>Study b</th>
<th>Composition</th>
<th>C_{11}, GPa</th>
<th>C_{12}, GPa</th>
<th>C_{44}, GPa</th>
<th>(\partial C_{11}/\partial P)_{TO}</th>
<th>(\partial C_{12}/\partial P)_{TO}</th>
<th>(\partial C_{44}/\partial P)_{TO}</th>
<th>P_{max}^*, GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Alm 92, Pyp, SpS, Grs 2</td>
<td>304.4(18)</td>
<td>110.3(15)</td>
<td>94.6(5)</td>
<td>6.6(3)</td>
<td>3.6(2)</td>
<td>1.3(1)</td>
<td>12</td>
</tr>
<tr>
<td>b</td>
<td>Alm 92, Pyp, SpS, Grs 2</td>
<td>304.8</td>
<td>112.3</td>
<td>94.4</td>
<td>7.15</td>
<td>3.85</td>
<td>1.29</td>
<td>0.5</td>
</tr>
<tr>
<td>c</td>
<td>Alm 92, Pyp, SpS, Grs 2</td>
<td>308.5</td>
<td>112.5</td>
<td>94.8</td>
<td>7.48</td>
<td>4.41</td>
<td>1.31</td>
<td>0.3</td>
</tr>
<tr>
<td>d</td>
<td>Alm 92, Pyp, SpS, Grs 2</td>
<td>306.2</td>
<td>112.5</td>
<td>92.7</td>
<td>7.48</td>
<td>4.41</td>
<td>1.31</td>
<td>0.3</td>
</tr>
<tr>
<td>e</td>
<td>Alm 92, Pyp, SpS, Grs 2</td>
<td>306.5</td>
<td>111.2</td>
<td>94.4</td>
<td>6.69</td>
<td>3.54</td>
<td>1.26</td>
<td>1</td>
</tr>
</tbody>
</table>

Numbers in parentheses are 1-σ deviations in last digit.

Studies are as follows: a, this study; b, Verma [1960]; c, Wang and Simmons [1974]; d, Soga [1967]; e, Isaak and Graham [1976].

[13] The robustness of our fitting results was confirmed on a standard sample of MgO and also demonstrated by low root mean square (RMS) differences between observed and calculated velocities (Table 2). Figures 2a and 2b show the observed velocities (symbols) and calculated velocity curves (lines) for grossular-rich and almandine-rich crystals at room and high pressure.

4. Discussion

4.1. Elastic Stiffness Constants, Aggregate Moduli, and Their Pressure Derivatives

The adiabatic aggregate bulk moduli, K_s, were fit to the Eulerian finite strain equation to obtain K_{SO} and K_{TO} = (\partial K_{SO}/\partial P)_{TO}. They were converted to isothermal values of K_{SO} and K_{TO} using equations (2) and (3). Pressure derivatives of individual elastic constants C_{ij} and the shear modulus, G, were obtained by fitting experimental values to the third-order finite strain equations [Davies, 1974]:

\[ C_{ijkl} = (1 + 2f)^{7/2} \left[ C_{ijkl}^{0} + \alpha_{1}f + \ldots \right] - P\Delta_{ijkl}. \]  

\[ \text{where, } f = 1/2[(\rho/\rho_{0})^{2/3} - 1] \text{ is the Eulerian finite strain, } C_{ijkl}^{0} \text{ is the value of elastic constant at ambient conditions, } \Delta_{ijkl} = -\delta_{ij}\delta_{kl} - \delta_{ik}\delta_{lj} - \delta_{il}\delta_{jk} \text{, and } P \text{ is pressure. } \alpha_{1} = 3K_{0}(\partial C_{ijkl}^{0}/\partial P + \Delta_{ijkl}) - 7C_{ijkl}^{0}/\partial P \text{ is the pressure derivative of the elastic constant at ambient conditions. The relation between the full notation of } C_{ijkl} \text{ and its contracted notation } C_{ij} \text{ is:} \]

\[ C_{ij} = 1/2\left[ (\rho/\rho_{0})^{2/3} - 1 \right] C_{ijkl}^{0}. \]

The fitted values of individual elastic constants, aggregate moduli, and their pressure derivatives are reported in Tables 4–7 together with values from the literature. Figures 3–6 show the plots of elastic constants, aggregate moduli and their pressure derivatives at ambient conditions and the anisotropy increases in magnitude modestly. For comparison, the ambient pressure anisotropy factor for other cubic minerals
such as MgO is 1.53 [Sinogeikin and Bass, 2000] and MgAl$_2$O$_4$ is 2.46 [Suzuki et al., 2000].

For our grossular-rich garnet, Grs$_{87}$And$_9$Pyp$_2$Alm$_1$, the fit value of the adiabatic bulk modulus $K_{S0}$ at ambient condition is 165.0 ± 0.9 GPa, which is about 3 GPa lower than previous values for grossular garnets [Bass, 1989; Isaak et al., 1992]. Assuming a linear dependence of the elastic moduli on composition expressed in mole fraction of the end members, correcting for the larger andradite content of our sample reduces this difference by about 1 GPa. For the shear modulus, our value also remains 1.4–3.0 GPa below the other studies after correction for andradite content. This level of uncertainty (2–3%) is typical of those found when comparing multiple measurements on the same or similar compositions for minerals such as fayalite [Isaak et al., 1993] and fluorite [Speziale and Duffy, 2002]. We note that the RMS deviations for the Brillouin measurements of Bass [1989] are about 50% larger than this study.

The fit values of $K_{T0}$ and $G_0$ of grossular are 3.9 ± 0.2 and 1.1 ± 0.1 and are compared with previous values in Table 5. Conrad et al. [1999] reported a Brillouin scattering study on grossular, andradite, and pyrope at high pressures using a very similar experimental technique as this study. In previous work [Jiang et al., 2004], we found large discrepancies between our results and those of Conrad et al. [1999] for andradite. For grossular, while the pressure derivative of the shear modulus is in excellent agreement, we obtain a much lower value for $K_{S0}$. Conrad et al. [1999] measured two crystal directions at each pressure compared with 36 in this study. The large differences may result from orientation errors in the previous study.

The high-pressure X-ray diffraction studies listed in Table 5 report a wide range of pressure derivatives. The determination of the bulk modulus and its pressure derivative from static compression relies on fits to the slope of the measured P-V curve, and hence is less direct than Brillouin scattering measurements. Static compression studies also suffer from a well-known tradeoff between fit values for $K_{T0}$ and $K_{T0}'$. Furthermore, if measurements are restricted to purely hydrostatic conditions, the compression range is very limited making it difficult to constrain the elastic moduli. On the other hand, data which covers a broad pressure range are subjected to variable degrees of nonhydrostatic stress depending on the nature and amount of the pressure transmitting medium. Figure 7 shows a comparison of the pressure-compression curve from our Brillouin data with previous X-ray diffraction studies. Below ~8 GPa, our
results are in very good agreement with earlier studies. The results of Pavese et al. [2001] and Zhang et al. [1999] indicated that their samples became increasingly less compressible than ours at higher pressures above 8 GPa. Olijnyk et al. [1991] performed compression studies on a synthetic grossular using both methanol-ethanol (M-E) and N2 as pressure transmitting media. Their P-V curve obtained using M-E as pressure medium is in very good agreement with ours to 10 GPa, while their P-V curve above 9 GPa derived using N2 as pressure medium deviates from ours. The methanol-ethanol mixture remains strictly hydrostatic up to 12 GPa, while N2 and Ne freeze above 3 GPa and 4.5 GPa, respectively, and generate nonhydrostatic stress conditions at higher pressures.

[19] For almandine-rich garnet Alm72Pyp20Sps3Grs3And2, both individual elastic constants Cij, aggregate moduli KS0 and G0 are in agreement (Tables 6 and 7) with those of a natural almandine with very similar composition Alm72Pyp23Grs3 [Soga, 1967], and they also agree well with other reported values (Table 7) after correcting for compositional differences. In general, there is a good agreement among various studies for ∂C44/∂P and (∂G/∂P). However, there is a wide range of reported values for the pressure derivatives of C11, C12, and KS0. Our result for KS0 is consistent with the low-end values of the range for KS0 in previous ultrasonic studies.

[20] A comparison of the isothermal compression curve derived from the Brillouin measurements with static X-ray diffraction data is shown in Figure 8. As with almandine, our compression curve agrees well with earlier compression studies at low pressure, and is also consistent until 10 GPa with data obtained using M-E as pressure transmitting medium [Sato et al., 1978]. Compression curves using Ne and He pressure medium [Zhang et al., 1999] deviate from ours above 6 GPa and 11 GPa, respectively. It is somewhat surprising (Figure 8) that X-ray data obtained with a He medium yields a similar compression curve to those obtained using Ne and NaCl since helium is expected to provide the closest approach to hydrostaticity. However, deviations from the hydrostatic compression curve can result for any pressure medium if an insufficient...
amount of the medium is loaded together with the sample, resulting in partial bridging and generation of deviatoric stresses. The large range of K$_0$ and especially K$_0$ values reported for X-ray studies in Tables 5 and 7 despite relatively modest differences in pressure-volume behavior exhibited in Figures 7 and 8 demonstrates the difficulty in reliably constraining these parameters from X-ray diffraction data.

4.2. Implications for High-Pressure Elasticity of Garnets

In general, ambient pressure elastic moduli determined by different techniques are in reasonable agreement (see Tables 5 and 7), but the pressure derivatives are highly discrepant as seen in Figure 9. Compression studies using X-ray diffraction can not give information about G and its pressure derivative, and provide less reliable value for K$_0$ and K$_0’$ because of parameter tradeoffs and nonhydrostatic stresses as discussed above. Ultrasonic methods provide a direct measurement of elasticity and acoustic velocities through travel time measurements, but are subject to uncertainties when polycrystalline samples are used, especially when combined with a limited pressure range [e.g., Wang and Ji, 2001]. The presence of pores and cracks in the sample could lead to an overestimate of pressure derivatives. Single-crystal ultrasonic studies were instead generally limited to very low pressures (<3 GPa). If the precision is adequate, slopes at lower pressures can be well constrained, but may not be applicable to higher pressures. Figure 9 shows pressure derivatives of garnets measured by Brillouin and ultrasonic methods. It is evident that pressure derivatives derived by ultrasonic methods are more scattered and in general higher than those by Brillouin scattering. The results of Conrad et al. [1999] probably suffered systematic errors, as discussed above and are not included. Brillouin scattering has advantages over other methods in that measurements on a large number of directions on high-quality single crystals can be performed under strictly hydrostatic conditions.

Table 8. End-Member Aggregate Elastic Properties of Garnets From Spectroscopic (Brillouin) Data

<table>
<thead>
<tr>
<th>Formula</th>
<th>a (Å)</th>
<th>K$_0$ (GPa)</th>
<th>G$_0$ (GPa)</th>
<th>(∂K$_0$/∂P)$_0$</th>
<th>(∂G/∂P)$_0$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Almandine</td>
<td>Fe$_3$Al$_2$Si$<em>3$O$</em>{12}$</td>
<td>11.531</td>
<td>175(2)</td>
<td>96(1)</td>
<td>4.9(2)</td>
<td>1.4(1)</td>
</tr>
<tr>
<td>Pyrope</td>
<td>Mg$_3$Al$_2$Si$<em>3$O$</em>{12}$</td>
<td>11.452</td>
<td>171(3)</td>
<td>94(2)</td>
<td>4.1(3)</td>
<td>1.3(2)</td>
</tr>
<tr>
<td>Grossular</td>
<td>Ca$_3$Al$_2$Si$<em>3$O$</em>{12}$</td>
<td>11.845</td>
<td>168(1)</td>
<td>109(4)</td>
<td>3.9(2)</td>
<td>1.1(1)</td>
</tr>
<tr>
<td>Andradite</td>
<td>Ca$_3$Fe$_2$Si$<em>3$O$</em>{12}$</td>
<td>12.058</td>
<td>157(2)</td>
<td>90(1)</td>
<td>4.7(1)</td>
<td>1.3(1)</td>
</tr>
<tr>
<td>Majorite</td>
<td>Mg$_3$Si$<em>3$O$</em>{12}$</td>
<td>11.494</td>
<td>166(3)</td>
<td>85(2)</td>
<td>4.2(3)</td>
<td>1.4(2)</td>
</tr>
</tbody>
</table>

aLattice parameters from Smyth and McCormick [1995].
bReferences are as follows: a, this study, extrapolated to end-member composition using a linear mixing model; b, Sinogeikin and Bass [2000]; c, Jiang et al. [2004]; d, Sinogeikin and Bass [2002].
from pyrope to garnet becomes 5.2% by 14 GPa. Ca$^2+$-Mg$^2+$
andradite series. The value of $K_0$
increase in $K_0$
dependence on mole fraction. It is notable that a similar
been extrapolated from our measured value assuming a linear
doing for garnets. The effect of Fe$^{2+}$-Mg$^{2+}$
is modest for garnets.

Figure 10. Bulk and shear moduli versus composition
for the pyrope-almandine series and the pyrope-grossular
series at selected pressures from 1 bar to 14 GPa using
the data of Table 8. Numbers next to each line are
pressures in GPa.

Table 8 provides values for the aggregate elastic
moduli of end-member garnets derived from this work and
other Brillouin scattering studies [Bass, 1989; Sinogeikin and
Bass, 2000, 2002; Chai et al., 1997; Jiang et al., 2004]. No
compositional trends are evident in the pressure derivatives
except for an increase in $K'_0$
upon substitution of Fe$^{3+}$ for
Mg$^{2+}$ in the pyrope-almandine series and a similar increase in
$K'_0$
upon substitution of Fe$^{3+}$ for Al$^{3+}$ in the grossular-
andradite series. The value of $K'_0$
for pure almandine has been extrapolated from our measured value assuming a linear
dependence on mole fraction. It is notable that a similar increase in $K'_0$
with Fe$^{3+}$-Mg$^{2+}$ substitution was observed for
the forsterite-fayalite series (S. Speziale et al., manuscript in
preparation, 2004).

Figure 10 shows the bulk and shear moduli versus
composition for the pyrope-almandine series and the pyrope-
grossular series at selected pressures from 0 to 14 GPa using
the data of Table 8 and assuming a linear dependence of both
moduli and their pressure derivatives on composition
expressed in mole fraction of the end members. At ambient
pressure, the bulk modulus varies by ~2% across the pyrope-
grossular and pyrope-almandine systems. Owing to the
higher $K'_0$
value for almandine, the increase in bulk modulus from
pyrope to garnet becomes 5.2% by 14 GPa. Ca$^{2+}$-Mg$^{2+}$
substitution has a large effect on the shear modulus: the
rigidity of grossular is 16% greater than pyrope at ambient
pressure. This is mostly maintained across the upper mantle
pressure interval as the shear modulus of grossular is 11%
greater than pyrope at 14 GPa. In contrast to other silicates
such as olivine, the effect of Fe$^{2+}$-Mg$^{2+}$ is modest for garnets.
The shear modulus of almandine is 2–3% larger than that of
pyrope at 0–14 GPa. The implications of these results for
assessing compositional variation in the upper mantle will be
the subject of a forthcoming publication.

5. Conclusions

Brillouin scattering measurements have been carried
out under hydrostatic conditions to 11 GPa on a grossular-
rich and an almandine-rich garnet. Individual, and aggregate
elastic moduli, and their pressure derivatives have been
determined by fitting Eulerian finite strain equations. All the
elastic constants, and aggregate moduli define nearly linear
trends with pressure. The adiabatic bulk, and Voigt-Reuss-
Hill average shear moduli and their pressure derivatives
were constrained to $K'_0 = 165.0 \pm 0.9$ GPa, $G'_0 = 104.2 \pm
0.3$ GPa, $K'_0 = 3.8 \pm 0.2$ and $G'_0 = 1.1 \pm 0.1$ for the
grossular-rich garnet and $K'_0 = 174.9 \pm 1.6$ GPa, $G'_0 =
95.6 \pm 0.5$ GPa, $K'_0 = 4.7 \pm 0.3$ and $G'_0 = 1.4 \pm 0.1$ for the
almandine-rich garnet.

A compression curve derived from our results is in
good agreement with high-pressure X-ray diffraction data,
especially those data recorded under hydrostatic conditions.
Elasticity data for almandine- and grossular-rich garnets
were compiled and evaluated. The ambient pressure moduli
show good agreement but reported pressure derivatives
exhibit wide variations that show little correlation with
composition, but rather appear to be related to experimental
technique. By restricting attention to high-pressure optical
spectroscopy (Brillouin scattering) studies, a set of self-
consistent aggregate elastic properties and their pressure
derivatives were derived for the major upper mantle garnet
end-member compositions.

These results (Table 8) demonstrate that pressure
derivatives of elastic moduli for mantle-relevant garnets fall
in a relatively narrow range and are considerably lower than
inferred from some earlier studies. Several recent mineral-
ological models for the upper mantle [Goes et al., 2000; Cammarano et al., 2003; Lee, 2003] have adopted elastic
properties for garnets, especially grossular garnets and Mg-
rich majorites outside the range of the optical spectroscopic
data set. Our development of an experimentally consistent
data set of end-member garnet properties will result in
improved mineralogical models for the mantle, and also
allow better constraints on the lateral variation of seismic
velocities due to local chemical variations in the mantle.

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